**MODERN COLLEGE OF ARTS,SCI. & COMM. PUNE-05.**

**DEPARTMENT OF STATISTICS.(Autonomous)**

**M.Sc.( I )- ST-15**

**EXPT.NO. 3 Date:**

**Title : Eigen value, Eigen vectors ,spectral decomposition and power of a matrix.**

1. Find all the eigen values and eigen vector corresponding to

any non zero characteristic root for the following matrix A and obtain 

(i)  ( ii )A= ****

2. Express the following matrix A as a linear combination of idempotent

matrices. Hence obtain **A3**

(i) A=**** (ii) 

(iii)  (iv) 

3.Find all the eigen values and eigen vector corresponding to

any non zero eigen values for the following matrix A



A=

4. Let matrix B be given by

B= 

Obtain all the eigen values of the matrix and left characteristic vector

corresponding to largest characteristic root. And right characteristic vectors

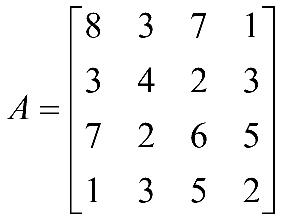
corresponding to remaining eigen values.

**\*\*\*\*\***

%%%%%%%%%%%%%%%%%%%%%%%%%% SOLUTION %%%%%%%%%%%%%%%%%%%%%%%

**Q1) 1. Find all the eigen values and eigen vector corresponding to**

**any   non zero characteristic root for the following matrix A and obtain **

1. 

>> A=[8 3 7 1;3 4 2 3;7 2 6 5;1 3 5 2]

A =

8 3 7 1

3 4 2 3

7 2 6 5

1 3 5 2

>> e=eig(A)

e =

-3.1694

2.3659

4.0000

16.8035

>> [V d]=eig(A)

V =

-0.3848 0.3306 -0.5774 0.6398

0.2699 0.7003 0.5774 0.3215

0.5919 -0.5134 -0.0000 0.6213

-0.6547 -0.3698 0.5774 0.3182

d =

-3.1694 0 0 0

0 2.3659 0 0

0 0 4.0000 0

0 0 0 16.8035

>> **Asq=V\*(d^2)\*(V)'**

% (V)’ because matrix is symmetric otherwise we use inverse

Asq =

123.0000 53.0000 109.0000 54.0000

53.0000 38.0000 56.0000 31.0000

109.0000 56.0000 114.0000 53.0000

54.0000 31.0000 53.0000 39.0000

**ii) A=**

>> A=[3 2 4;2 4 -2;4 -2 5]

A =

3 2 4

2 4 -2

4 -2 5

>> e=eig(A)

e =

-1.5470

5.3866

8.1603

>> [V d]=eig(A)

V =

0.6929 0.4244 0.5829

-0.4523 0.8854 -0.1070

-0.5615 -0.1895 0.8055

d =

-1.5470 0 0

0 5.3866 0

0 0 8.1603

**>> Asq=V\*(d^2)\*(V)'**

Asq =

29.0000 6.0000 28.0000

6.0000 24.0000 -10.0000

28.0000 -10.0000 45.000

**Q2) Express the following matrix A as a linear combination of idempotent**

**matrices. Hence obtain A3**

**i)**

>> A=[1 2 -1;2 4 -2;4 -4 5]

A

1 2 -1

2 4 -2

4 -4 5

>> [v1 d1]=eig(A)

v1 =

0.3714 -0.3333 -0.3333

-0.5571 -0.6667 -0.6667

-0.7428 -0.6667 0.6667

d1 =

-0.0000 0 0

0 3.0000 0

0 0 7.0000

>> [v2 d2]=eig(A')

v2 =

-0.1849 0.8944 0.8944

0.7396 -0.4472 -0.0000

-0.6472 0.0000 0.4472

d2 =

7.0000 0 0

0 0.0000 0

0 0 3.0000

>> E1=v1(:,1)/(v2(:,2)'\*v1(:,1))

E1 =

0.6389

-0.9583

-1.2778

>> E2=v1(:,2)/(v2(:,3)'\*v1(:,2))

E2 =

0.5590

1.1180

1.1180

>> E3=v1(:,3)/(v2(:,1)'\*v1(:,3))

E3 =

0.3863

0.7726

-0.7726

>> a1=E1\*v2(:,2)'

a1 =

0.5714 -0.2857 0.0000

-0.8571 0.4286 -0.0000

-1.1429 0.5714 -0.0000

>> a2=E2\*v2(:,3)'

a2 =

0.5000 -0.0000 0.2500

1.0000 -0.0000 0.5000

1.0000 -0.0000 0.5000

>> a3=E3\*v2(:,1)'

a3 =

-0.0714 0.2857 -0.2500

-0.1429 0.5714 -0.5000

0.1429 -0.5714 0.5000

>> Spec\_D=(d1(1,1)\*a1)+(d1(2,2)\*a2)+(d1(3,3)\*a3)

Spec\_D =

1.0000 2.0000 -1.0000

2.0000 4.0000 -2.0000

4.0000 -4.0000 5.0000

**% we observe D=A**

>> SD3=(d1(1,1)^3\*a1)+(d1(2,2)^3\*a2)+(d1(3,3)^3\*a3)

SD3 =

-11.0000 98.0000 -79.0000

-22.0000 196.0000 -158.0000

76.0000 -196.0000 185.0000

**ii)**

>> A=[1 2 -1;3 4 2;1 5 3]

A =

1 2 -1

3 4 2

1 5 3

>> [v1 d1]=eig(A)

v1 =

0.0721 0.5605 0.6750

0.6092 -0.0161 -0.5574

0.7898 -0.8280 0.4835

d1 =

6.9478 0 0

0 2.4201 0

0 0 -1.3679

>> [v2 d2]=eig(A')

v2 =

0.6288 -0.7378 0.4693

-0.6424 -0.5003 0.8299

0.4381 0.4532 0.3016

d2 =

-1.3679 0 0

0 2.4201 0

0 0 6.9478

>> E1=v1(:,1)/(v2(:,3)'\*v1(:,1))

E1 =

0.0927

0.7834

1.0157

>> E2=v1(:,2)/(v2(:,2)'\*v1(:,2))

E2 =

-0.7179

0.0206

1.0606

>> E3=v1(:,3)/(v2(:,1)'\*v1(:,3))

E3 =

0.6788

-0.5606

0.4863

>> a1=E1\*v2(:,3)'

a1 =

0.0435 0.0769 0.0279

0.3677 0.6502 0.2363

0.4767 0.8430 0.3063

>> a2=E2\*v2(:,2)'

a2 =

0.5296 0.3592 -0.3253

-0.0152 -0.0103 0.0093

-0.7825 -0.5306 0.4807

>> a3=E3\*v2(:,1)'

a3 =

0.4269 -0.4361 0.2974

-0.3525 0.3601 -0.2456

0.3058 -0.3124 0.2130

>> Spec\_D=(d1(1,1)\*a1)+(d1(2,2)\*a2)+(d1(3,3)\*a3)

Spec\_D =

1.0000 2.0000 -1.0000

3.0000 4.0000 2.0000

1.0000 5.0000 3.0000

>> SD3=(d1(1,1)^3\*a1)+(d1(2,2)^3\*a2)+(d1(3,3)^3\*a3)

SD3 =

21.0000 32.0000 4.0000

124.0000 217.0000 80.0000

148.0000 276.0000 109.0000

**(iii) **

>> A=[1 2 -1;2 4 -2;-1 -2 3]

A =

1 2 -1

2 4 -2

-1 -2 3

>> %% Matrix A is symmetric hence we get same eigen values and same eigen vectors for A and A'

>> [v,d]=eig(A)

v =

0.8944 -0.2433 0.3753

-0.4472 -0.4865 0.7505

0 -0.8391 -0.5439

d =

-0.0000 0 0

0 1.5505 0

0 0 6.4495

>> SD=(d(1,1)\*v(:,1)\*v(:,1)')+(d(2,2)\*v(:,2)\*v(:,2)')+(d(3,3)\*v(:,3)\*v(:,3)')

SD =

1.0000 2.0000 -1.0000

2.0000 4.0000 -2.0000

-1.0000 -2.0000 3.0000

**>> %%Finding A^3 using above formula we get,**

>> A3=(d(1,1)^3\*v(:,1)\*v(:,1)')+(d(2,2)^3\*v(:,2)\*v(:,2)')+(d(3,3)^3\*v(:,3)\*v(:,3)')

A3=

38.0000 76.0000 -54.0000

76.0000 152.0000 -108.0000

-54.0000 -108.0000 82.0000

**iv)**

>> D=[15 -6 -18 -6;-4 5 8 4;12 -6 -15 -6;4 -2 -8 -1]

D =

15 -6 -18 -6

-4 5 8 4

12 -6 -15 -6

4 -2 -8 -1

>> e=eig(D)

e =

-3.0000

1.0000

3.0000

3.0000

>> e(1,1)

ans =

-3.0000

>> X1=null(A-(e(1,1)\*eye(4)),'r')

X1 =

1.0000

-1.0000

1.0000

1.0000

>> X2=null(A-(e(2,1)\*eye(4)),'r')

X2 =

-0.0000

-1.0000

-0.0000

1.0000

>> X3=null(D-(e(3,1)\*eye(4)),'r')

X3 =

0.5000 -1.0000

1.0000 0

0 -1.0000

0 1.0000

>> X=[X1 X2 X3]

X =

1.0000 -0.0000 0.5000 -1.0000

-1.0000 -1.0000 1.0000 0

1.0000 -0.0000 0 -1.0000

1.0000 1.0000 0 1.0000

>> [v d]=eig(A)

v =

0.5000 0.0000 -0.6539 0.6022

-0.5000 0.7071 -0.4761 0.0814

0.5000 0.0000 -0.4158 0.5616

0.5000 -0.7071 0.4158 -0.5616

D =

-3.0000 0 0 0

0 1.0000 0 0

0 0 3.0000 0

0 0 0 3.0000

>> SpecD=X\*d\*(inv(X))

SpecD =

15.0000 -6.0000 -18.0000 -6.0000

-4.0000 5.0000 8.0000 4.0000

12.0000 -6.0000 -15.0000 -6.0000

4.0000 -2.0000 -8.0000 -1.0000

>> SD3=X\*(d^3)\*(inv(X))

SD3 =

135.0000 -54.0000 -162.0000 -54.0000

-4.0000 29.0000 32.0000 28.0000

108.0000 -54.0000 -135.0000 -54.0000

4.0000 -2.0000 -32.0000 -1.0000

**Q3) Find all the  eigen values and eigen vector corresponding to**

**any  non zero eigen values for the following matrix A**

>> A=[6 3 -8;0 -2 0;1 0 -3]

A =

6 3 -8

0 -2 0

1 0 -3

>> e=eig(A)

e =

5

-2

-2

>> [v d]=eig(A)

v =

0.9923 0.7071 0.7071

0 0 0.0000

0.1240 0.7071 0.7071

d =

5 0 0

0 -2 0

0 0 -2

**Q4)    Let matrix B be given by**

**Obtain all the eigen values of the matrix and left characteristic vector**

**corresponding to largest characteristic root. And right characteristic vectors**

**corresponding to remaining eigen values.**

>>B=[2 2 0;2 1 1;-7 2 -3]

B =

2 2 0

2 1 1

-7 2 -3

>> e=eig(B)

e =

-4.0000

3.0000

1.0000

>> [v1 d1]=eig(B)

v1 =

0.0747 -0.6667 -0.4364

-0.2242 -0.3333 0.2182

0.9717 0.6667 0.8729

d1 =

-4.0000 0 0

0 3.0000 0

0 0 1.0000

>> [v2 d2]=eig(B')

v2 =

-0.6350 0.2357 0.7276

-0.7620 -0.9428 -0.4851

-0.1270 -0.2357 0.4851

d2 =

3.0000 0 0

0 1.0000 0

0 0 -4.0000

**>> %%% Left characheristics vector corresponding to the largest characteristics root will be**

>> LCV=v2(:,1)

LCV =

-0.6350

-0.7620

-0.1270

**>> %%%Right characteristics vectors corresponding to remaining eigen values**

>> RCV1=v1(:,1)

RCV1 =

0.0747

-0.2242

0.9717

>> RCV2=v1(:,3)

RCV2 =

-0.4364

0.2182

0.8729

**>> RCV=[RCV1 RCV2]**

**RCV =**

**0.0747 -0.4364**

**-0.2242 0.2182**

**0.9717 0.8729**